Decoherence measure by gravitational wave interferometers

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We consider the possibility to measure the quantum decoherence using gravitational wave interferometers. Gravitational wave interferometers create the superposition state of photons and measure the interference of the photon state. If the decoherence occurs, the interference of the photon state vanishes and it can be measured by the interferometers. As examples of decoherence mechanisms, we consider 1) decoherence by spontaneous localization, 2) gravitational decoherence and 3) decoherence by extra-dimensional gravity.

I. INTRODUCTION

Gravitational wave interferometers, such as LIGO[1], VIRGO[2], TAMA[3] and GEO600[4], are designed to detect gravitational waves directly. By gravitational wave detection, we expect to gain better understanding of the relativistic theory of gravity. They are also expected to serve as a new kind of telescope to observe our universe. With the anticipation of high sensitivity of interferometers in next several years, new astronomy using gravitational-wave telescopes has been developed recently.[6]

In this paper, we propose alternative experiments possible with interferometers. Gravitational wave interferometers are huge macroscopic devices, yet, their measurement sensitivity is near the standard quantum limit by cutting-edge optical technology. Because of this high sensitivity, gravitational wave interferometers may be able to test Quantum Physics. Here, we focus on the quantum decoherence. As we discuss in detail below, the quantum decoherence is considered to be a key to solve the measurement problem of Quantum Physics. If we can measure that the quantum decoherence is occurring at a fundamental level, it will give an important clue to solving the measurement problem and will have major impact on the foundation of Quantum Physics.

Because of the high sensitivity near the standard quantum limit, one may think that the quantization of the test mass is crucial to study the interferometer output. However, the test mass quantization is not relevant because its effect appears only in low frequency (around several *Hz*) and is normally filtered out from the data processing of the interferometer output.[7] This limits the measurement possibility of the quantum decoherence acting on the test mass. Unless the quantum decoherence occurs quite frequently (such as by several hundreds *Hz*), one cannot read out its effect from the interferometer output. An idea was proposed to measure the decoherence effect on the test mass using the interferometer technology in Ref.[8]. The possibility to measure a certain decoherence mechanism acting on the test mass of the gravitational wave interferometers was also discussed in Ref.[9, 10]

Here, instead of the test mass, we consider the quantum decoherence of photons. There are a couple of advantages to measure the photon decoherence using the gravitational wave interferometers;

- Gravitational wave interferometers are made to measure the photon interference and they can directly measure the photon decoherence.
- The photon is a fundamental particle and one can measure the fundamental decoherence.
- For high sensitivity, the gravitational wave interferometers use high power laser. This will statistically increase the possibility of measuring decoherence.
- The operation of the gravitational wave interferometers is very stable. For example, LIGO succeeded a year-long operation recently. This also helps statistically increasing the possibility of measuring decoherence.
- As we discuss in Sec.III, the decoherence measurement is shot-noise limited at relatively higher sideband frequency. In this region, many mechanical noise sources have already been identified and well-studied. This may serve a clean experimental setup.
- The decoherence effect of the test mass can be seen as the noise of gravitational wave interferometers, therefore, it is hard to identify whether the decoherence actually occurs. On the other hand, as we show in Sec.III, by

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changing the carrier light for the homodyne detection, one can distinguish the photon decoherence and the noise of the interferometers.

The advanced design of gravitational wave interferometers uses the quantum non-demolition technology (hereafter QND) to measure the photon interference.[11] In Sec.II and Sec.III, we discuss how the QND gravitational wave interferometers can measure the photon decoherence and its measurement limit. In Sec.IV and Sec.V, we present some specific proposed models of the fundamental decoherence and discuss the measurement limit of the model parameters. In Sec.VI, we conclude this paper.

II. INTERFEROMETRIC MEASUREMENT OF QUANTUM DECOHERENCE

Gravitational wave interferometers have two arm cavities. Let us call two arm cavities by R-arm and L-arm. This huge device creates a superposition state from an injected photon, namely the superposition of the photon in the R-arm and the photon in the L-arm. By measuring the interference of these photon states, one is able to extract the information from gravitational waves.

We denote the superposition state of the injected photon by

$$|\gamma_{in}>= (|\gamma_R> + |\gamma_L>)/\sqrt{2}\,, \tag{2.1}$$

where $|\gamma_{R/L}>$ is the photon state in the R/L-arm. The interferometer has two output ports to be measured. One is called by the bright port and its state is described by

$$|\gamma_b>= (|\gamma_R> + |\gamma_L>)/\sqrt{2}\,. \tag{2.2}$$

The other is the dark port which is

$$|\gamma_d>= (|\gamma_R> - |\gamma_L>)/\sqrt{2}\,. \tag{2.3}$$

If the superposition state of the photon has no external coupling, it is apparent that we will only see the bright port state. More specifically, the probability of measuring the photon coming out from the bright port is given as $P_b = |<\gamma_b|\gamma_{in}>|^2 = 1$ and the probability to measure the photon from the dark port is given as $P_d = |<\gamma_d|\gamma_{in}>|^2 = 0$.

Let us assume that the superposition state of the photon is coupled to an external field ϕ while the photon state evolves in the interferometer. The photon in the R/L-arm induces the excitation of the external field and we can formally write the total state by

$$(|\gamma_R>\otimes|\phi_R> + |\gamma_L>\otimes|\phi_L>)/\sqrt{2}\,, \tag{2.4}$$

where the states $|\phi_{R/L}>$ describe the external fields that couples with the photon in the R/L-arm and we ignore the back-action to the photon due to the field coupling. Taking the trace over the field state, the quantum state of the photon is described by the reduced density matrix as

$$\begin{aligned}\rho_\gamma &= |\gamma_R><\gamma_R|/2 + |\gamma_L><\gamma_L|/2 \\ &+|\gamma_R><\gamma_L|\times<\phi_L|\phi_R>/2 + |\gamma_L><\gamma_R|\times<\phi_R|\phi_L>/2\,,\end{aligned} \tag{2.5}$$

where we use $<\phi_R|\phi_R>=<\phi_L|\phi_L>=1$. When $Re[<\phi_R|\phi_L>] = 0$, the interference term vanishes and the quantum state becomes effectively the classical sum over the R-state and the L-state. By this situation, we say the superposition state of the photon is decohered due to the field coupling.

The probabilities to measure the photon coming from the bright port and from the dark port are given by

$$P_b = <\gamma_b|\rho_\gamma|\gamma_b>= \frac{1}{2} + \frac{1}{2}Re[<\phi_R|\phi_L>]\,, \tag{2.6}$$

$$P_d = <\gamma_d|\rho_\gamma|\gamma_d>= \frac{1}{2} - \frac{1}{2}Re[<\phi_R|\phi_L>]\,. \tag{2.7}$$

If $Re[<\phi_R|\phi_L>] = 1$, the photon comes out of the bright port only. On the other hand, if $Re[<\phi_R|\phi_L>] = 0$, the photon comes out of both the bright and dark ports with equal probability. Thus, the quantum decoherence of the superposition state can easily be observed by measuring the dark port of the interferometer.

III. QND LASER INTERFEROMETER

In the previous section, we consider the ideal case that we inject a single photon into the interferometer and see the outcome of the dark port. In the actual operation of gravitational wave interferometers, we continuously inject the photon and the interferometer output is processed in the frequency domain. In this section, we discuss how we can see the photon decoherence and the measurement limit of the decoherence probability.

We denote the injected light power by I. and the probability of the decoherence by P_{deco}. It is apparent that the decoherence of the photon state leads to the continuous light output from the dark port of the power $< I_{deco} >= IP_{deco}/2$ on average. However, one cannot measure this continuous component of the light output in the data analysis process of gravitational-wave detection because it uses the frequency decomposition of the light output with respect to the sideband Ω and the continuous component is filtered out of the data. Instead, we consider to measure the statistical fluctuation of the light output due to the decoherence.

The number of photons injected to the interferometer in a short time interval Δt is $N = I\Delta t/(\hbar\omega_0)$ where ω_0 is the frequency of the carrier light. The statistical distribution of the number n of the decohered photon state in this time interval follows the Poisson distribution $P[n] = e^{-P_{deco}N}\frac{(P_{deco}N)^n}{n!}$. It is easy to see that the statistical average of the decohered photon state number is $< n >= \sum_n nP[n] = P_{deco}N$ and the averaged light power from the dark port becomes

$$< I_{deco} >= \frac{\hbar\omega_0}{2\Delta t} < n >= I\frac{P_{deco}}{2}\,. \tag{3.1}$$

The statistical fluctuation of the decohered photon state number is $< (n- < n >)^2 >= P_{deco}N$ and the fluctuation of the laser power we can measure from the dark port averaged over the time interval Δt is derived as

$$< \Delta_{\Delta t} I_{deco}^2 >= \left(\frac{\hbar\omega_0}{2\Delta t}\right)^2 < (n- < n >)^2 >= I\frac{\hbar\omega_0}{\Delta t}\frac{P_{deco}}{4}\,. \tag{3.2}$$

We suppose that the decoherence process is spontaneous and that decoherence events do not correlate with each other, in this case the output from the dark port becomes the white spectrum with respect to the sideband frequency. The two-time correlation of the light power fluctuation is given by

$$< \Delta I_{deco}(t)\Delta I_{deco}(t') >= \frac{\hbar\omega_0 I P_{deco}}{4} \times \delta(t-t')\,. \tag{3.3}$$

The spectral density of the light power from the dark port $S_{I,deco}(\Omega)$ is the Fourier transformation of the two-time correlation defined by $< \Delta I_{deco}(t)\Delta I_{deco}(t') >= \int (d\Omega/2\pi) S_{I,deco}(\Omega)e^{i\Omega(t-t')}$ and we have

$$S_{I,deco}(\Omega) = \frac{\hbar\omega_0 I P_{deco}}{4}\,. \tag{3.4}$$

If this is larger than the noise spectrum density, we may be able to observe the decoherence. In order to see the theoretical limit for the decoherence measurement and we consider an ideal QND interferometer without any mechanical noise.

1) Decoherence Measurement by the QND interferometer

For gravitational wave detection, we use the so-called homodyne detection, where we measure a particular output quadrature field $E(t)$ by superposing the carrier light of the power $I_{carrier}$. The noise spectrum of the homodyne detection is given as (A17) in Appendix. It is apparent that the noise spectrum becomes smaller for the smaller carrier light because the superposed carrier light amplifies the noise, but it does not amplify the decoherence signal. For the fixed carrier light power, one can lower the noise due to the opto-mechanical coupling by choosing the carrier light phase as $\alpha = 0$ and we have the white noise spectrum as

$$S_{I,homo}(\Omega) \;=\; \frac{\hbar\omega_0 I_{carrier}}{4}\,. \tag{3.5}$$

This is because the quadrature field we measure is purely the amplitude quadrature which does not couple to the test mass location. Therefore, the noise does not include the back-action noise and it has only the shot noise which is a spontaneous process.

Since the decoherence signal (3.4) and the noise (3.5) have the same spectrum shape, one cannot distinguish the decoherence signal from the noise for a fixed carrier light power. By changing the carrier light power, it is easy to extract the decoherence signal and that is the unique advantage in measuring the photon decoherence.

The decoherence measurement is the shot-noise limited and its theoretical limit is simply given by

$$P_{deco} \;>\; \frac{I_{carrier}}{I}\,. \tag{3.6}$$

With the current LIGO operation, the noise curve reaches the fraction of the theoretical noise. The carrier light superposed onto the output quadrature field is around $I_{carrier} \approx 0.001I$, therefore, even the successful operation of the the current gravitational wave interferometer constraints the decoherence probability by

$$P_{deco} \leq 0.001\,. \tag{3.7}$$

2) zero-carrier light

Because the homodyne detection amplifies the noise of the interferometer it is more advantageous for a decoherence measurement to directly measure the output light power by the photodetector without superposing the carrier light.

The noise spectrum of the direct photo detection is given by (A25) in Appendix. For simplicity, we consider the theoretical limit at specific sideband frequencies, $\Omega = 0, \gamma$, and the frequency, $\gamma << \Omega << (\omega_0^2\gamma)^{1/3}$. The theoretical limit is derived as

$$P_{deco} \;>\; \begin{cases} \frac{\hbar\omega_0\gamma}{I_{SQL}}\left(10\frac{I}{I_{SQL}} + \frac{429}{4}\left(\frac{I}{I_{SQL}}\right)^3\right) & \text{when } \Omega = 0, \\ \frac{\hbar\omega_0\gamma}{I_{SQL}}\left(\frac{29}{5}\frac{I}{I_{SQL}} + \frac{29}{125}\left(\frac{I}{I_{SQL}}\right)^3\right) & \text{when } \Omega = \gamma, \\ 5\frac{\hbar\omega_0\gamma}{I_{SQL}}\frac{I}{I_{SQL}} & \text{when } \gamma << \Omega << (\omega_0^2\gamma)^{1/3}. \end{cases}, \tag{3.8}$$

which suggests that high-frequency data give us the better theoretical limit for the decoherence measurement. It is interesting to point out that the noise has stronger dependence on the injected light power I and that we may not be able to measure the decoherence if the injected light is too strong.

Suppose we inject the SQL light power to a LIGO-scale interferometer, $I = I_{SQL} = 1.0\times10^4 W = 1.0\times10^{11} erg/sec$, $\omega_0 = 1.8\times10^{15} sec^{-1}$ and $\gamma = 2\pi\times100 sec^{-1}$, we have the theoretical limit for the decoherence measurement as

$$P_{deco} > 6.0\times10^{-20}\,. \tag{3.9}$$

IV. DECOHERENCE BY SPONTANEOUS LOCALIZATION

Quantum Physics is based on two distinct processes. One is the unitary evolution process of the state, which is linear, deterministic and reversible. The other is according to *Copenhagen interpretation* the state reduction process at the time of measuring the quantum state, which is non-linear, stochastic and irreversible. These processes are incompatible and the identification of the borderline between the regimes these two processes to be applied is called by *the measurement problem*. Because quantum measurement involves a macroscopic measurement device with a huge number of degrees of freedom interacting with surrounding environment, it is far from trivial to identify when the state reduction process would take over the dynamical process of Quantum Physics.

The paradoxical nature of this measurement problem is best illustrated by the famous Schrödinger's cat;[12] a cat in a box is evolving into linear superposition of the quantum state where the cat is 'alive' and the state where the cat is 'dead'. Now we open the box and measure the cat. Suddenly the superposition state disappears and we will see the cat is either alive or dead with nonzero classical probability. We then ask; Why do we see only the state of the living cat or the state of the dead cat? Why don't we see the superposition state of the living cat and the dead cat? What dynamical process transforms the quantum superposition into the classical sum of these specific states?

One of proposals solving this issue is to unify these two physical processes by adding small non-linear and stochastic effects to the unitary evolution process. This new stochastic effects are made to be small for a microscopic system so that we still have the unitary evolution, but, the effect becomes large for a macroscopic system. When a system is measured by a macroscopic measurement device, the stochastic effects strongly act on the device and the state reduction process occurs dynamically to the quantum state of the device.

Along this idea, several models have been proposed to modify the Schrödinger equations which govern the unitary evolution process of the quantum state. One of the simplest models is called Quantum Mechanics with Spontaneous Localization (hereafter QMSL). In this model, we specifically consider the quantum evolution of a collection of particles. Most of the time, the wave-function evolves by the usual Schrödinger equations, but, spontaneously at a mean rate λ, the localization process occurs following a probability distribution derived from the wave-function.

By the localization process, the localization operator is applied to the wave-function which localizes the wave packet of each particle by the length scale $1/\sqrt{\alpha}$. This process collapses the wave-function dynamically in the position-representation without measurement. An interesting feature of QMSL is that, although the localization operator localizes the wave packet of each particle by the rate λ, it cumulatively localizes the reduced density matrix of the center of mass by the rate $N\lambda$ where $N$is the number of the particles. This means that, for a microscopic system, the localization effect becomes negligible with sufficiently small λ, but, it could be substantially large for a macroscopic system because of extremely large N. According to Ref.[13], it was suggested that the model parameters could be

$$\lambda \simeq 10^{-16}[s^{-1}]\,, \quad 1/\sqrt{\alpha} \simeq 10^{-5}[cm]\,. \tag{4.1}$$

More thorough survey of various experiments[10] indicates that

$$\lambda \simeq 10^{-7\pm2}[s^{-1}]\,, \quad 1/\sqrt{\alpha} \simeq 10^{-5}[cm]\,. \tag{4.2}$$

Let us suppose that the photon dynamics has a similar spontaneous localization process. Gravitational wave interferomters are sufficiently large compared to the localization scale $1/\sqrt{\alpha}$, thus, once the spontaneous localization occurs to the photon propagating the detectors, the photon state becomes the classical sum of the photon state in the R-arm and the photon state in the L-arm, that is, the photon state becomes completely decohered. As we discussed in Sec.II, this can be measured by the photo-detection at the dark port. The decoherence probability is the mean rate of the spontaneous localization multiplied by the photon propagation time as

$$P_{deco} = \lambda/\gamma\,. \tag{4.3}$$

For a LIGO-scale interferometer with the suggested parameters of QMSL (4.1), we have the decoherence probability as

$$P_{deco} \sim 10^{-19}\,. \tag{4.4}$$

If we use the parameters (4.2), the decoherence probability would be

$$P_{deco} \sim 10^{-10\pm2}\,. \tag{4.5}$$

These values are consistent with the fact that the current gravitational wave detectors do not see the decoherence effect by the homodyne detection since it satisfies (3.7). However, if one can prepare the ideal dark port with the zero-carrier light, one may be possibly measure the decoherence due to the spontaneous localization according to (3.9).

V. GRAVITATIONAL DECOHERENCE - SEMI-CLASSICAL GRAVITY

It was suggested that gravity might serve the mechanism for the fundamental decoherence.[14] In this section, we estimate the decoherence probability due to the gravitational coupling of the interferometer using the semi-classical gravity.

Let us denote the probability of the graviton emission by P_g for the photon in either the R-arm or in the L-arm. By the interferometer, we have the photon superposition state, $(|\gamma_R> + |\gamma_L>)/\sqrt{2}$. Due to the gravitational coupling, the photon state evolves into

$$|\gamma_R>/\sqrt{2}\otimes(|0> + c_R|g_R>)/\sqrt{1+P_g} + |\gamma_L>/\sqrt{2}\otimes(|0> + c_L|g_L>)/\sqrt{1+P_g}\,, \tag{5.1}$$

where $|0>$ is the zero-graviton state and $|g_R>$ and $|g_L>$ are the one-graviton state emitted from the photon in the R-arm and the L-arm of the interferometer. For simplicity, we here ignore the back-action to the photon due to the graviton emission. c_R and c_L are the amplitudes of the graviton emission and we have $|c_R|^2 = |c_L|^2 = P_g$. We suppose that the scale of the interferometer is large enough so that there is no interference between the one-graviton states emitted from the R-arm and the L-arm, i.e. $< g_R|g_L > \approx 0$. Then the reduced density matrix for the photon state can be written as

$$\begin{aligned} &\frac{1}{2}(|\gamma_R><\gamma_R| + |\gamma_L><\gamma_L|) \\ &+\frac{1}{2(1+P_g)}(|\gamma_R><\gamma_L| + |\gamma_L><\gamma_R|)\,. \end{aligned} \tag{5.2}$$

This shows that the decoherence probability of the photon state is approximately equal to the graviton emission probability, $P_{deco} \simeq P_g$.

In the following, we calculate the emission probability using the semi-classical gravity. We are especially interested in the observational possibility to distinct some gravitational models and we consider the standard 4-D gravity, and the brane gravity.

Standard 4-D gravity

Photons that propagate freely do not emit gravitons. When the photon hits the mirrors of the arm cavity, the orbit of the photon moves from a null geodesic to another by exchange its momentum with the mirror, and it emits gravitons.

Although the photon is a relativistic particle, we use the quadrupole formula to estimate the emission probability of graviton. The energy flux is given by

$$L = \frac{G}{5c^5} \left| \frac{d^3 \hat{\mathbf{I}}}{dt^3} \right|^2 , \tag{5.3}$$

where $\hat{\mathbf{I}}$ is the trace-free component of the mass quadrupole moment tensor of the system. When the photon hits the mirror, they exchange the momentum and produce the nontrivial quadrupole. This process occurs in the mirror's surface layer which has the thickness of the photon wavelength in the time scale of the photon frequency. Thus, we assume $d^3\mathbf{I}/dt^3 \approx \hbar\omega_0^2$ and that the duration of the graviton emission is $\approx 1/\omega_0$. Diving the energy flux by the graviton energy ($\approx \hbar\omega_0$), the number of the emitted gravitons is estimated as

$$N \approx \frac{G\hbar\omega_0^2}{c^5} . \tag{5.4}$$

We regard N as the probability of the graviton emission because $N << 1$. The emission probability is multiplied by the finesse $1/T$ because the photon is trapped by the highly reflective mirrors of the cavity (T is the mirrors' transmissivity of the cavity.) The total probability of the decoherence is obtained as

$$P_{deco} \approx \frac{G\hbar\omega_0^2}{c^5 T} . \tag{5.5}$$

For the LIGO-scale interferometer, this becomes

$$P_{deco} \approx 10^{-55} , \tag{5.6}$$

thus, it is unlikely to measure the gravitational decoherence.

Brane gravity

Brane cosmology was suggested as an interesting solution of the hierarchical problem in particle physics. In the brane cosmology, spacetime is $(4+1)$-dimensional but, matters are located only on a $(3+1)$-hypersurface (3-brane). Because of the extra-dimensional freedom of the gravitational field, it has an extra channel to emit gravitons, which is called by the Kaluza-Klein mode (hereafter, KK-mode). The size of the extra-dimension (ℓ) is not well-constrained by the present experiments of gravitational forces if it is smaller than $0.1mm$.[15] If the characteristic wavelength of graviton emission is smaller than the size of the extra-dimension, we may expect huge emission of KK-mode gravitons.

The modified quadrupole formula was derived[16] and the energy flux through the KK-modes is given as

$$L_{KK} = -\frac{ell^2}{18c^2} \frac{d^5 I}{dt^5} , \tag{5.7}$$

where I is the trace of the mass quadrupole. The probability of the KK-mode graviton emission is estimted as

$$P_{KK-deco} \approx \frac{\ell^2 \omega_0^2}{c^2 T} . \tag{5.8}$$

For the LIGO-scale interferometer, this becomes

$$P_{KK-deco} \approx \left(\frac{\ell}{0.1\mathrm{mm}} \right)^2 10^7 . \tag{5.9}$$

This successful operation of the current LIGO implies that $P_{KK-deco} < 0.001$, which constrains the size of the extra-dimension to

$$\ell < 10^{-6}\text{mm}\,. \tag{5.10}$$

With the zero-carrier light limit, the gravitational wave detectors can measure the gravitational decoherence due to the KK-mode graviton emission if

$$\ell > 10^{-14}\text{mm}\,. \tag{5.11}$$

VI. CONCLUSION

The difficulty in measuring a specific decoherence process is that there is not way to separate the decoherence signal and the noise from the measurement output. Both the decoherence and the noise usually appear in the measurement output in the quite similar way because they come from the system's coupling with uncontrolled environment. As we see from Sec.III, this is actually the case when we consider the decoherence measurement using interferometers, i.e. the decoherence signal and the quantum noise have the common spectrum shape. However, the quantum noise is proportional to the light power of the carrier light, thus, it is possible to separate the noise and the decoherence signal by changing the carrier light. This allow us to estimate the measurement limit of the decoherence probability and we obtain the following results:

- The current successful operation of LIGO shows that the decoherence probability must be

$$P_{deco} \leq 0.001\,. \tag{6.1}$$

- By taking the zero carrier-light limit, LIGO-scale interferometers can measure the decoherence if the decoherence probability is

$$P_{deco} > 6.0 \times 10^{-20}\,. \tag{6.2}$$

We consider two kinds of the decoherence mechanism. One is the spontaneous localization as the modification of the Schrödinger equation and it has two model parameters. The proposed value of these parameters are just consistent with the current operation of LIGO, but, if we could take the zero carrier-light limit, it is possible to see the fundamental decoherence due to this mechanism, otherwise, we could experimentally deny this modification of Quantum Physics.

We also consider gravitational decoherence. For standard 4D semi-classical gravity, it is almost impossible to see the gravitational decoherence of photons. If we consider a extra-dimensional gravity model, one could see the strong decoherence due to the coupling with the bulk gravitational field. We show that the brane world gravity is such a case and we find that the size of the extra-dimension must be constrained by

$$\ell < 10^{-6}\,, \tag{6.3}$$

by the current operation of the LIGO experiment.

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APPENDIX A: QUANTUM DESCRIPTION OF QND LASER INTERFEROMETER

The standard (single-mode) expression of the quantized 1-dimensional optical field is given as

$$E \;=\; \int_0^\infty \frac{d\omega}{2\pi}\sqrt{\frac{4\pi\hbar\omega}{Ac}}\left(ae^{-i\omega t} + a^\dagger e^{i\omega t}\right)\,, \tag{A1}$$

where A is the cross section area of the beam. a and $a^\dagger$ are the annihilation and creation operators that satisfy the commutation

$$[a(\omega), a(\omega')] = 0\,, \quad [a(\omega), a^\dagger(\omega')] = 2\pi\delta(\omega - \omega')\,. \tag{A2}$$

We are interested in the optical fields of the sideband $\omega = \omega_0 + \Omega$ about the carrier light frequency ω_0. In this case, the optical field is conveniently described by the two-photon formalism as

$$E = E^{(c)}\cos(\omega_0 t) + E^{(s)}\sin(\omega_0 t)\,, \tag{A3}$$

$$E^{(c)} = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\int_0^\infty \frac{d\Omega}{2\pi}\left(a^{(c)}e^{-i\Omega t} + a^{(c)\dagger}e^{i\Omega t}\right)\,, \tag{A4}$$

$$E^{(s)} = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\int_0^\infty \frac{d\Omega}{2\pi}\left(a^{(s)}e^{-i\Omega t} + a^{(s)\dagger}e^{i\Omega t}\right)\,, \tag{A5}$$

where Ω is the sideband frequency and we only consider the domain where $\Omega << \omega_0$. $E^{(c)}$ and E_s are called by the cosine and sine quadrature fields. $a^{(c)}(\Omega)$ and $a^{(s)}(\Omega)$ are the quantum operators and are related to the creation/annihilation operators of the single-photon mode by

$$a^{(c)}(\Omega) = \frac{1}{\sqrt{2}}\left(a(\omega_0+\Omega) + a^\dagger(\omega_0-\Omega)\right)\,, \quad a^{(s)}(\Omega) = \frac{1}{i\sqrt{2}}\left(a(\omega_0+\Omega) - a^\dagger(\omega_0-\Omega)\right)\,. \tag{A6}$$

The interferometer is an optical device to produce the poderomotive squeezing of the optical fields through its arm cavities. With the general expression of the optical field in the two-photon formalism, we use the quantum operators $a^{(c)}_{in}$ and $a^{(s)}_{in}$ in the place of $a^{(c)}$ and $a^{(s)}$ for the input optical field and we use the quantum operators $a^{(c)}_{out}$ and $a^{(s)}_{out}$ for the output optical field. For an ideal interferometer, these operators are related by

$$a^{(c)}_{out} = a^{(c)}_{in}e^{2i\beta}\,, \quad a^{(s)}_{out} = \left(a^{(s)}_{in} - \mathcal{K}a^{(c)}_{in}\right)e^{2i\beta} + \sqrt{2\mathcal{K}}\frac{h}{h_{SQL}}e^{i\beta}\,, \tag{A7}$$

where we use $\beta = \arctan(\Omega/\gamma)$ as the phase shift of off-resonant optical fields, $\mathcal{K} = 2(I/I_{SQL})\gamma^4/\Omega^2/(\Omega^2+\gamma^2)$ as the opto-mechanical coupling constant, $I_{SQL} = mL^4\gamma^4/(4\omega_0)$ as the laser light power to reach the standard quantum limit, and $h_{SQL} = \sqrt{8\hbar/(m\Omega^2L^2)}$ as the standard quantum limit for the gravitational-wave measurement. (γ is the cavities' half bandwidths, m is the reduced test mass of the interferometer and L is the arm length.) h is the Fourier transform of the gravitational-wave signal $h(t)$ defined by

$$h = \int_{-\infty}^{\infty} dt e^{i\Omega t}h(t)\,, \tag{A8}$$

and the gravitational-wave signal can be read out from the sine quadrature of the output field.

1) Homodyne detection

By the homodyne detection, it is possible to measure the output field by the direct photo detection. With the beam splitter, we superpose the laser light of the carrier frequency ω_0 on the output field

$$E = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\sqrt{2}D\cos(\omega_0 t + \alpha)\,, \tag{A9}$$

where the light power is $\hbar\omega_0 D^2$. The resulting optical field for the homodyne detection is written as

$$E_{homo} = E^{(c)}_{homo}\cos(\omega_0 t) + E^{(s)}_{homo}\sin(\omega_0 t)\,, \tag{A10}$$

$$E^{(c)}_{homo} = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\left[D\cos(\alpha) + \frac{1}{\sqrt{2}}\int_0^\infty \frac{d\Omega}{2\pi}\left(a^{(c)}_{out}e^{-i\Omega t} + a^{(c)}_{out}{}^\dagger e^{i\Omega t}\right)\right]\,, \tag{A11}$$

$$E^{(s)}_{homo} = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\left[-D\sin(\alpha) + \frac{1}{\sqrt{2}}\int_0^\infty \frac{d\Omega}{2\pi}\left(a^{(s)}_{out}e^{-i\Omega t} + a^{(s)}_{out}{}^\dagger e^{i\Omega t}\right)\right]\,. \tag{A12}$$

The light power to be detected by the photodetector becomes

$$\begin{aligned} I_{homo}(t) &= \frac{E^2_{homo}}{4\pi}Ac \\ &\approx \hbar\omega_0\left[D^2 + \frac{D\cos(\alpha)}{\sqrt{2}}\int_0^\infty \frac{d\Omega}{2\pi}\left(a^{(c)}_{out}e^{-i\Omega t} + a^{(c)}_{out}{}^\dagger e^{i\Omega t}\right) - \frac{D\sin(\alpha)}{\sqrt{2}}\int_0^\infty \frac{d\Omega}{2\pi}\left(a^{(s)}_{out}e^{-i\Omega t} + a^{(s)}_{out}{}^\dagger e^{i\Omega t}\right)\right], \end{aligned} \tag{A13}$$

where we ignore the very high frequency mode.

For the conventional interferometer, the input quantum state is the vacuum state with respect to the in-state quantum operator, i.e. $a(\omega)|0>=0$. Using (A7), we obtain the expectation value of the detected light power,

$$
\begin{aligned}
< I_{homo} > (t) &= \hbar\omega_0 \left[D^2 - D\sin(\alpha) \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \sqrt{\mathcal{K}} \frac{h}{h_{SQL}} e^{i\beta - i\Omega t} \right], && \text{(A14)} \\
< I_{homo} > (\Omega) &:= \int dt e^{i\Omega t} < I_{homo} > (t) = -D\sin(\alpha)\hbar\omega_0 \sqrt{\mathcal{K}} \frac{h}{h_{SQL}} e^{i\beta}. && \text{(A15)}
\end{aligned}
$$

The quantum fluctuation of the light power $\delta I_{homo} = I_{homo} - < I_{homo} >$ becomes

$$
\begin{aligned}
< \delta I_{homo}(t) \delta I_{homo}(t') > &= \int \frac{d\Omega}{2\pi} S_{I,homo}(\Omega) e^{-i\Omega(t-t')}, && \text{(A16)} \\
S_{I,homo}(\Omega) &= \frac{(\hbar\omega_0)^2}{4} D^2 \left\{ (\cos(\alpha) + \mathcal{K}\sin(\alpha))^2 + \sin^2(\alpha) \right\}, && \text{(A17)}
\end{aligned}
$$

where we use

$$
\begin{aligned}
& < a_{out}^{(c)}(\Omega) a_{out}^{(c)\dagger}(\Omega') > = < a_{out}^{(c)\dagger}(\Omega) a_{out}^{(c)}(\Omega') > = \pi\delta(\Omega - \Omega'), && \text{(A18)} \\
& < a_{out}^{(c)}(\Omega) a_{out}^{(s)\dagger}(\Omega') > = < a_{out}^{(c)\dagger}(\Omega) a_{out}^{(s)}(\Omega') > = \pi(i - \mathcal{K})\delta(\Omega - \Omega'), && \text{(A19)} \\
& < a_{out}^{(s)}(\Omega) a_{out}^{(c)\dagger}(\Omega') > = < a_{out}^{(s)\dagger}(\Omega) a_{out}^{(c)}(\Omega') > = \pi(-i - \mathcal{K})\delta(\Omega - \Omega'), && \text{(A20)} \\
& < a_{out}^{(s)}(\Omega) a_{out}^{(s)\dagger}(\Omega') > = < a_{out}^{(s)\dagger}(\Omega) a_{out}^{(s)}(\Omega') > = \pi(1 + \mathcal{K}^2)\delta(\Omega - \Omega'). && \text{(A21)}
\end{aligned}
$$

2) Direct detection

The homodyne detection is advantageous for the gravitational wave measurement because the signal is amplified by the carrier light. However, that is not the case for the decoherence measurement because the quantum noise is also amplified. We therefore consider the detection at the zero-carrier-light limit. In this case, the light power to be detected by the photodetector becomes

$$
I_{dir}(t) \approx \frac{\hbar\omega_0}{2} \left[\left\{ \int \frac{d\Omega}{2\pi} \left(a_{out}^{(c)} e^{-i\Omega t} + a_{out}^{(c)\dagger} e^{i\Omega t} \right) \right\}^2 + \left\{ \int_0^{\infty} \frac{d\Omega}{2\pi} \left(a_{out}^{(s)} e^{-i\Omega t} + a_{out}^{(s)\dagger} e^{i\Omega t} \right) \right\}^2 \right]. \qquad \text{(A22)}
$$

The expectational value of the light power with the vacuum input state is obtained as

$$
\begin{aligned}
< I_{dir}(t) > &\approx \hbar\omega_0 \left\{ \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \sqrt{\mathcal{K}} \frac{h}{h_{SQL}} e^{i\beta} e^{-i\Omega t} \right\}^2 + \frac{\hbar\omega_0}{4} \int \frac{d\Omega}{2\pi} \mathcal{K}^2 \\
&= \hbar\omega_0 \left\{ \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \sqrt{\mathcal{K}} \frac{h}{h_{SQL}} e^{i\beta} e^{-i\Omega t} \right\}^2 + \frac{5}{4} \hbar\omega_0 \left(\frac{I}{I_{SQL}} \right)^2 \gamma. && \text{(A23)}
\end{aligned}
$$

The second term is due to the poderomotive squeezing of the interferometer and we subtract the vacuum contribution for regularization. The integration of $\mathcal{K}^2$ over Ω becomes infinite for small $|\Omega|$ since $\mathcal{K} \propto 1/\Omega^2$. This low-frequency behavior comes from the free motion of the mirror. We regulate the divergence by using $\mathcal{K} = 2(I/I_{SQL})\gamma^4/(\Omega - i\epsilon)^2/(\Omega^2 + \gamma^2)$, which corresponds to adding an infinitesimally small damping force to the mirror motion.

The quantum fluctuation of the light power is obtained as

$$
\begin{aligned}
< \{\delta I_{dir}(t), \delta I_{dir}(t')\} > &\approx \frac{(\hbar\omega_0)^2}{8} \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\Omega'}{2\pi} \Big\{ 1 + (i - \mathcal{K}(\Omega))(i - \mathcal{K}(\Omega')) + (-i - \mathcal{K}(\Omega))(-i - \mathcal{K}(\Omega')) \\
&\qquad + \left(1 + \mathcal{K}^2(\Omega)\right)\left(1 + \mathcal{K}^2(\Omega')\right) \Big\} e^{-i(\Omega + \Omega')(t-t')} \\
&= \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} S_{I,dir}(\Omega) e^{-i\Omega(t-t')}, && \text{(A24)} \\
S_{I,dir}(\Omega) &\approx \frac{(\hbar\omega_0)^2}{16} \int_{-\infty}^{\infty} \frac{d\bar{\Omega}}{2\pi} \left\{ (\mathcal{K}(\Omega_+) + \mathcal{K}(\Omega_-))^2 + \mathcal{K}^2(\Omega_+)\mathcal{K}^2(\Omega_-) \right\}_{\Omega_\pm = (\Omega \pm \bar{\Omega})/2}
\end{aligned}
$$

$$= (\hbar\omega_0)^2 \Bigg\{ \frac{1}{4}\left(\frac{I}{I_{SQL}}\right)^2 \left\{ 5\gamma + \frac{4\gamma^5(5\gamma^2-\Omega^2)}{(\Omega^2+\gamma^2)^2(\Omega^2+4\gamma^2)} \right\}$$
$$+16\left(\frac{I}{I_{SQL}}\right)^4 \frac{\gamma^9(1716\gamma^8-715\gamma^6\Omega^2-741\gamma^4\Omega^4-33\gamma^2\Omega^6+5\Omega^8)}{(\Omega^2+\gamma^2)^5(\Omega^2+4\gamma^2)^3} \Bigg\}. \tag{A25}$$

We note that the Ω-integration here is consistent with the approximation of the two-photon formalism, $|\Omega| < \omega_0$ because $\mathcal{K}$ becomes small for large $|\Omega|$.

The noise spectrum (A25) does not include the shot noise because it comes entirely from the opto-mechanical coupling. In the case of the homodyne detection, the quantum fluctuation of the output quadrature field around the carrier light frequency, $\omega \approx \omega_0$ is converted into the fluctuation at the sideband frequency by the superposition of the carrier light. On the other hand, there is no such conversion process for the direct detection. The shot noise comes from the fluctuation at the low frequency, $\omega \approx 0 << \omega_0$ in the one-photon formalism, which is not properly described by the two-photon formalism. Hence, it is reasonable that the noise spectrum (A25) does not include the shot noise.

The shot noise to be added to (A25) can be derived from the one-photon formalism. Because it comes from the quantum fluctuation of the input vacuum, we can use the standard form (A1) and we have

$$< \{\delta I_{dir}(t), \delta I_{dir}(t')\} >_{shot} = \int_0^\infty \frac{d\omega}{2\pi}\int_0^\infty \frac{d\omega'}{2\pi} 2\hbar^2(\omega\omega')\cos\left((\omega+\omega')(t-t')\right)$$
$$= \int_{-\infty}^\infty \frac{d\Omega}{2\pi} S_{I,shot}(\Omega) e^{-i\Omega(t-t')}\,, \tag{A26}$$
$$S_{I,shot}(\Omega) = \frac{\hbar^2\Omega^3}{6\pi}\,. \tag{A27}$$

The shot noise is small for $\Omega < (\omega_0^2\gamma)^{1/3}$ and we ignore this noise in Sec.III.

[1] http://www.ligo.caltech.edu/
[2] http://www.virgo.infn.it/
[3] http://tamago.mtk.nao.ac.jp/
[4] http://geo600.aei.mpg.de/
[5] Some of the recent studies are reported by L. Blanchet, L. P. Grishchuk, G. Schaefer, summary of PPN1 and PPN2 parallel sessions at MG11 (Berlin, August,2006) [arXiv:gr-qc/0703034].
[6] For example, S. A. Hughes, Annals Phys. 303 (2003) 142 [arXiv:astro-ph/0210481], and references therein.
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